Gas Turbine Cycle

Lecture 4
Enhancements of gas cycle

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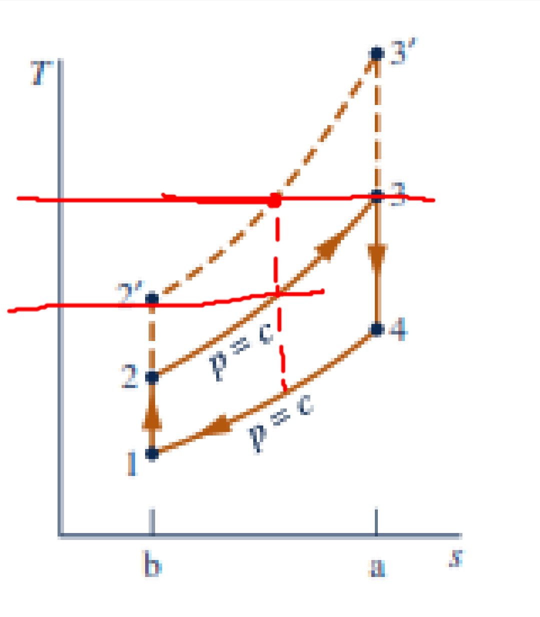
Effect of pressure ratio on performance:

In the figure beside, the increase in the pressure ratio changes the cycle from (1,2,3,4,1) to (1,2́,3́,4,1). Since the average temp of heat addition is greater in the later cycle and both cycles have the same heat rejection process, then cycle (1,2́,3́,4,1) would have the greater thermal efficiency.
This can also be approved as follow:



- for cost C_p , k (cold air- standard)

$$\eta = \frac{C_p(T_3-T_4)-C_p(T_2-T_1)}{C_p(T_3-T_2)} = 1-\frac{T_4-T_1}{T_3-T_2}$$

for further arrangement:

$$\eta = 1-\frac{T_1(\frac{T_4}{T_1}-1)}{T_2(\frac{T_3}{T_2}-1)}$$

Effect of pressure ratio on performance:

From previous relations:

$$r_p^{\frac{k-1}{k}} = \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

or

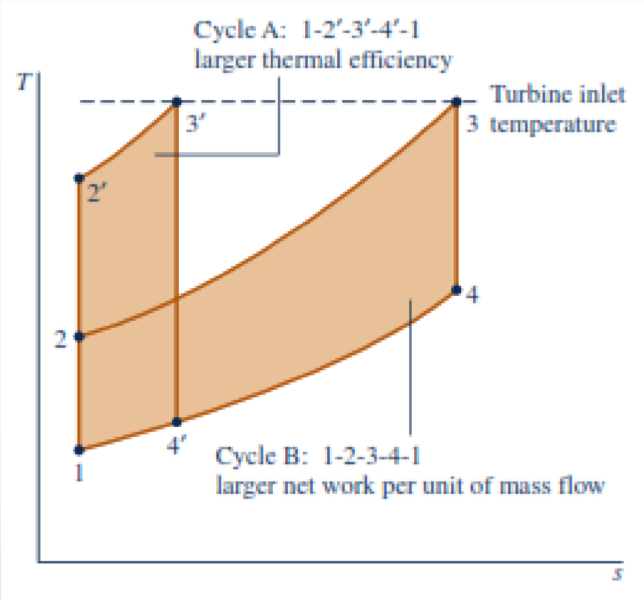
$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

So

$$\eta = 1 - \frac{T_1}{T_2}$$

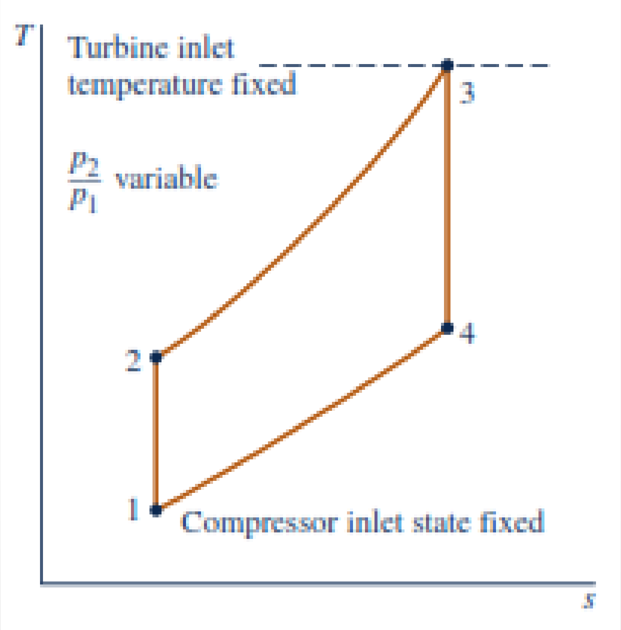
And

$$\eta = 1 - \frac{1}{\frac{T_2}{T_1}} = 1 - \frac{1}{\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}}$$



Compressor pressure ratio for maximum net work

- maximum net work occurs at minimum compressor work.



The net work of the cycle per unit flow is:

$$\frac{\dot{W}_{cycle}}{\dot{m}} = w_{net} = (h_3 - h_4) - (h_2 - h_1)$$

Assume constant C_p :

$$W_{net} = C_p\,[(T_3 - T_4) - (T_2 - T_1)]$$

Rearrange the above equation, we get:

$$W_{net} = C_p\,T_1\left[\frac{T_3}{T_1} - \frac{T_4 T_3}{T_3 T_1} - \frac{T_2}{T_1} + 1\right]$$

Compressor pressure ratio for maximum pet work

Replacing the temp ratios $\frac{T_2}{T_1}$ *and* $\frac{T_4}{T_3}$ *by*:

$$\frac{T_2}{T_1} = (\frac{P_2}{P_1})^{\frac{k-1}{k}} \quad , \frac{T_4}{T_3} = (\frac{P_1}{P_2})^{\frac{k-1}{k}}$$

$$W_{net} = C_p\ T_1[\frac{T_3}{T_1} - \frac{T_4 T_3}{T_3 T_1} - \frac{T_2}{T_1} + 1]$$

$$W_{net} = C_p\ T_1[\frac{T_3}{T_1} - \frac{T_3}{T_1}(\frac{P_1}{P_2})^{\frac{k-1}{k}} - (\frac{P_2}{P_1})^{\frac{k-1}{k}} + 1]$$

This equation shows that for specified value of T_1, T_3, and C_p, the value of the net work output per unit mass flow varies with the pressure ratio $\frac{p_2}{p_1}$ only.

To determine the pressure ratio that maximized the net work output per unit mass flow, first form derivative:

$$\frac{\partial w_{net}}{\partial(\frac{P_2}{P_1})} = C_p T_1 \frac{k-1}{k}[\left(\frac{T_3}{T_1}\right)\left(\frac{P_1}{P_2}\right)^{\frac{-1}{k}}\left(\frac{P_1}{P_2}\right)^2 - \left(\frac{P_2}{P_1}\right)^{\frac{-1}{k}}]$$

$$= C_p T_1 \frac{k-1}{k}[\left(\frac{T_3}{T_1}\right)\left(\frac{P_1}{P_2}\right)^{\frac{2k-1}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{-1}{k}}]$$

Compressor pressure ratio for maximum pet work

When the derivative is set to zero, we get :

$$\left(\frac{T_3}{T_1}\right)\left(\frac{P_1}{P_2}\right)^{\frac{2k-1}{k}} - \left(\frac{P_2}{P_1}\right)^{\frac{-1}{k}} = 0$$

$$\therefore \left(\frac{T_3}{T_1}\right)\left(\frac{P_1}{P_2}\right)^{\frac{2k-1}{k}} = \left(\frac{P_2}{P_1}\right)^{\frac{-1}{k}}$$

$$\frac{T_3}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{-1}{k}}\left(\frac{1}{\left(\frac{P_1}{P_2}\right)^{\frac{2k-1}{k}}}\right)$$

$$\frac{T_3}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{2k-2}{k}}$$

or

$$\frac{P_2}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2k-2}}$$